

state space

1) Representation

2) Analysis

3) Design

→ For Linear systems:-

$$\dot{x} = Ax + Bu \Rightarrow F(x, u)$$

$$y = Cx + \underbrace{Du}_0$$

$A, B, C, D \rightsquigarrow$ system matrices

→ In discrete form

$$x((k+1)T) = A_d x(kT) + B_d u(kT)$$

$$y(kT) = C_d x(kT) + D_d u(kT)$$

$$A_d \neq A ; C_d \neq C ; D_d \neq D ; B_d \neq B$$

$x \rightarrow$ state vector

$y \rightarrow$ output

$u \rightarrow$ Control input

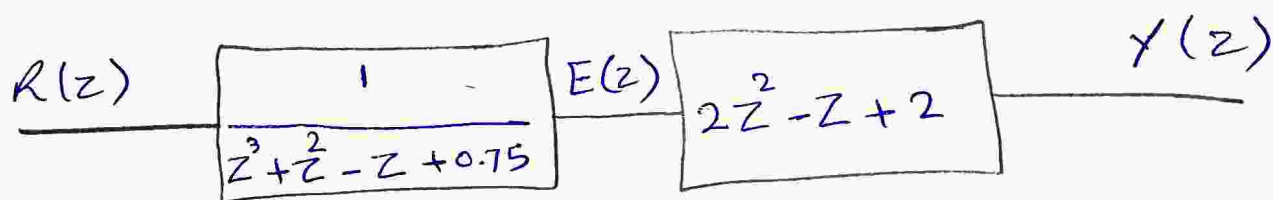
* Representation of state variable Model:-

- 1) Controller Canonical Form.
- 2) observer Canonical Form.
- 3) Parallel / Diagonal Form.
- 4) Cascaded Form.

Ex Given $\frac{Y(z)}{R(z)} = \frac{2z^2 - z + 2}{z^3 + z^2 - z + 0.75}$

- Required
- 1) Controller Canonical
 - 2) observer Canonical

1) Controller



1st Block

$$\frac{E(z)}{R(z)} = \frac{1}{z^3 + z^2 - z + 0.75}$$

$$z^3 E(z) + z^2 E(z) - z E(z) + 0.75 E(z) = R(z)$$

Inverse of Z.T

$$e(k+3) + e(k+2) - e(k+1) + 0.75e(k) = r(k)$$

$$\text{Let: } e(k) = x_1 \rightarrow e(k+1) = x_1(k+1)$$

$$e(k+1) = x_2 \rightarrow e(k+2) = x_2(k+1)$$

$$e(k+2) = x_3 \rightarrow e(k+3) = x_3(k+1)$$

$$e(k+3) = r(k) - e(k+2) + e(k+1) - 0.75e(k)$$

$$x_1(k+1) = x_2(k)$$

$$x_2(k+1) = x_3(k)$$

$$x_3(k+1) = r(k) - x_3(k) + x_2(k) - 0.75x_1(k)$$

$$X(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.75 & 1 & -1 \end{bmatrix} X(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(k)$$

لأنه عكس الإشارة

لاحظ الجزء الثاني عليه مربع

$$\frac{Y(z)}{E(z)} = 2z^2 - z + 2$$

$$Y(z) = 2z^2 E(z) - z E(z) + 2 E(z)$$

↓
Z⁻¹.T

$$y(k) = 2e(k+2) - e(k+1) + 2e(k)$$

Assumption

$$x_1(k) = e(k)$$

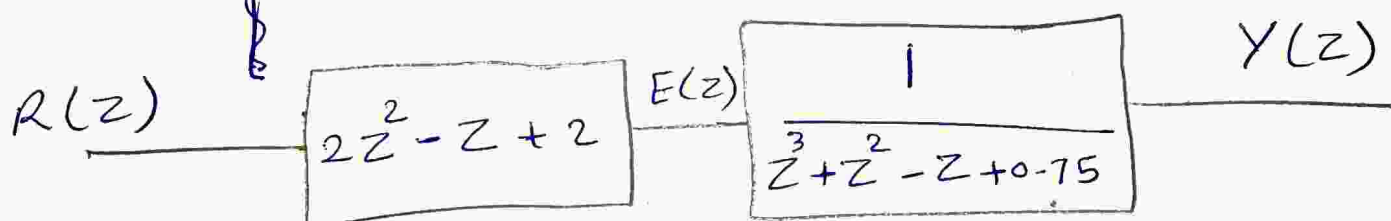
$$x_2(k) = e(k+1)$$

$$x_3(k) = e(k+2)$$

$$y = 2x_3(k) - x_2(k) + 2x_1(k)$$

$$y(k) = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} x(k)$$

b) observer Canonical form



1st block

$$\frac{E(z)}{R(z)} = 2z^2 - z + 2$$

$$E(z) = 2z^2 R(z) - z R(z) + 2R(z) \quad \downarrow \quad \vec{z} \cdot \vec{r}$$

$$e(k) = 2r(k+2) - r(k+1) + 2r(k) \rightarrow \textcircled{1}$$

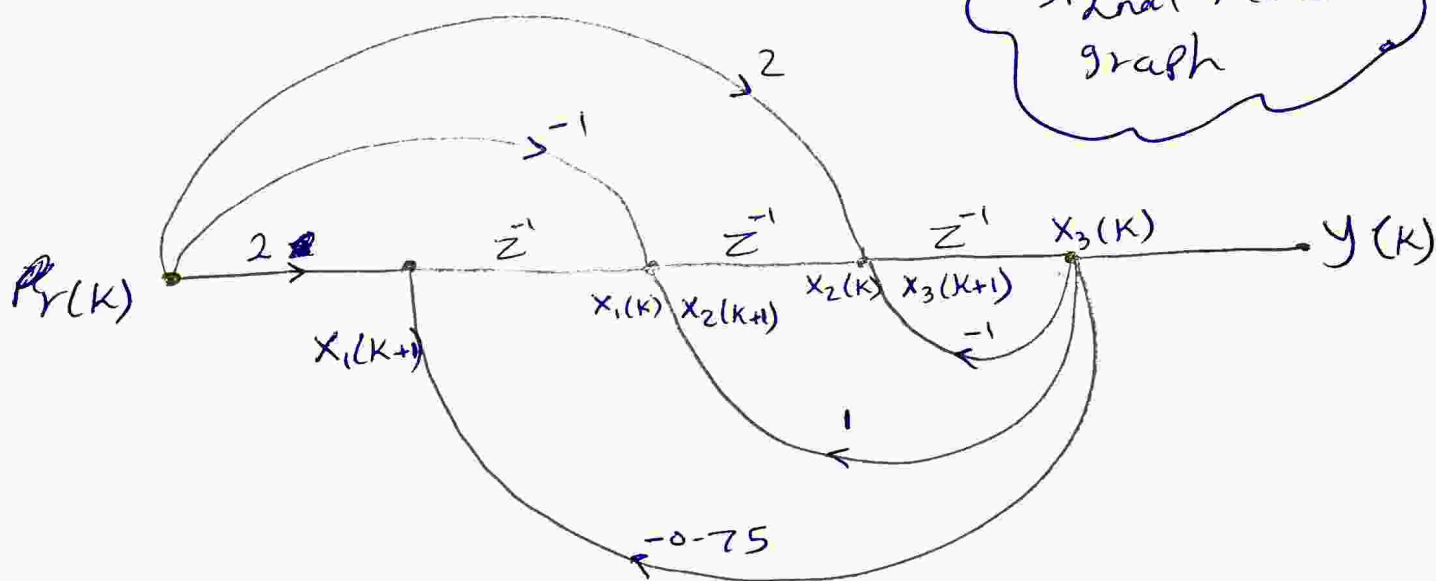
$$\frac{Y(z)}{R(z)} = \frac{2z^2 - z + 2}{z^3 + z^2 - z + 0.75}$$

$$\times \frac{z^{-3}}{z^{-3}}$$

$$= \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 + z^{-1} - z^{-2} + 0.75z^{-3}}$$

$$= \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 - [-z^{-1} + z^{-2} - 0.75z^{-3}]}$$

signal flow graph



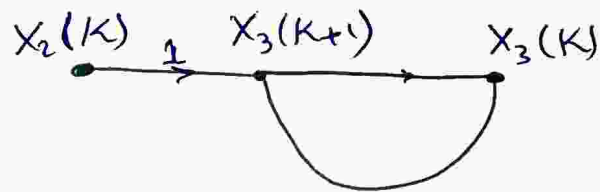
$$x_1(k+1) = -0.75 x_3(k) + 2 R(k)$$

$$x_2(k+1) = x_1(k) - R(k) + x_3(k)$$

$$x_3(k+1) = x_2(k) + 2R(k) - x_3(k)$$

→ في الرسم السابق مثلاً نقطة $X_3(k+1)$ ، $X_2(k)$ بينهم

$z_{\text{ain}} = 1$ لكنهم مش نقطة واحدة



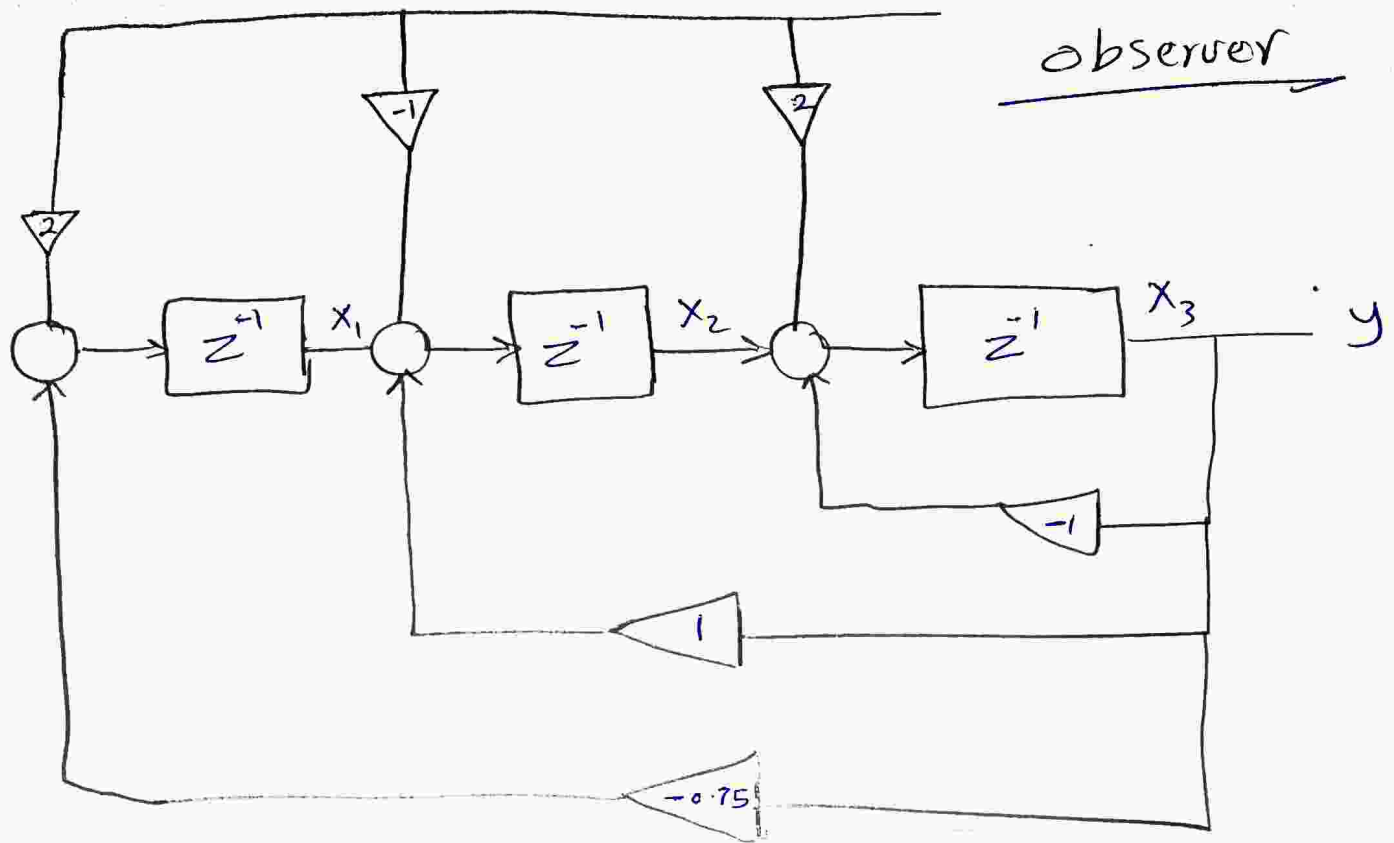
$$X(k+1) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} X(k) + \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} r(k)$$

ماتريكس الحالة

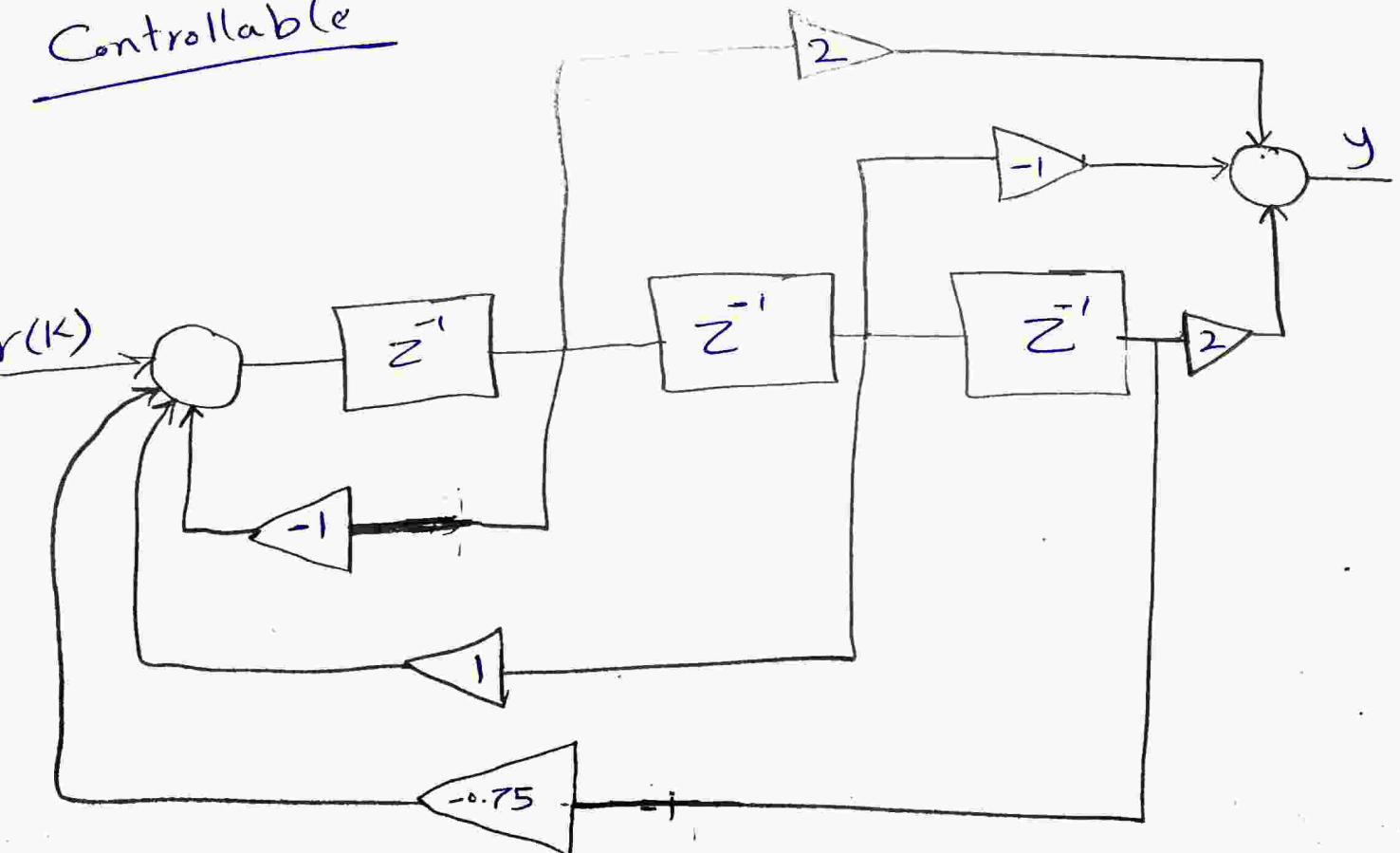
$$y(k) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X(k)$$

$$\frac{Y(z)}{R(z)} = \frac{\sum P_i}{1 - [\text{loops}]}$$

$$= \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 - [-z^{-1} + z^{-2} - 0.75z^{-3}]}$$



Controllable



Given T.F

$$\frac{Y(z)}{R(z)} = \frac{z + 0.5}{(z-1)(z-0.1)(z+0.4)}$$

Representation in Diagonal Form:-

$$\frac{Y(z)}{R(z)} = \frac{A}{z-1} + \frac{B}{z-0.1} + \frac{C}{z+0.4}$$

$$= \frac{1.19}{z-1} - \frac{1.333}{z-0.1} + \frac{0.143}{z+0.4}$$

$$Y(z) = \underbrace{\frac{1.19}{z-1} R(z)}_{Y_1(z)} + \underbrace{\frac{-1.333}{z-0.1} R(z)}_{Y_2(z)} + \underbrace{\frac{0.143}{z+0.4} R(z)}_{Y_3(z)}$$

$$Y_1(z) = \frac{1.19}{z-1} R(z) \Rightarrow zX_1(z) - X_1(z) = 1.19 R(z)$$

$$y_1(k+1) - y_1(k) = 1.19 r(k)$$

$$\text{Let } y_1(k) = x_1(k) \Rightarrow x_1(k+1) = y_1(k+1)$$

$$\boxed{x_1(k+1) = x_1(k) + 1.19 r(k)} \rightarrow \textcircled{1}$$

$$Y_2(z) = \frac{-1.333}{z-0.1} R(z)$$

$$z Y_2(z) - 0.1 Y_2(z) = -1.33 R(z)$$

$$y_2(k+1) - 0.1 y_2(k) = -1.33 r(k)$$

$$\text{Let } x_2(k) = y_2(k) \Rightarrow x_2(k+1) = y_2(k+1)$$

$$x_2(k+1) = 0.1 x_2(k) - 1.33 r(k) \rightarrow \textcircled{2}$$

$$Y_3(z) = \frac{0.143}{z+0.4} R(z)$$

$$z Y_3(z) + 0.4 Y_3(z) = 0.143 R(z)$$

$$x_3(k+1) = -0.4 x_3(k) + 0.143 r(k) \rightarrow \textcircled{3}$$

$$X(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -0.4 \end{bmatrix} + \begin{bmatrix} 1.19 \\ -1.33 \\ 0.143 \end{bmatrix} r(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} X(k)$$

